

FUZZY SYSTEM IDENTIFICATION: A FIREFLY OPTIMIZATION APPROACH

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ABSTRACT

Soft Computing based algorithms are very important way of designing knowledge based systems which are comparatively large and highly complex. Designing a system for the academic evaluation of a university or an institution of higher learning is a complex task as it involves large number of parameters to be considered which are difficult to measure. In this paper fuzzy logic based system for academic rating of institutions of higher learning is designed using firefly (FA) optimization approach. A fuzzy model identification problem is formulated as minimization problem and the model is identified by applying FA optimization based approach. All the input parameters and their membership functions along with the consequents for the rules of the rule base are modified randomly to find the desired values for the system with minimum MSE. Here we have taken 14 inputs each with four MFs and 136 Rules. The performance of the FA is compared with that of BB-BC and Parallel BB-BC [1, 2, and 3] based optimization approaches.

KEYWORDS: Firefly Optimization Approach, Fuzzy Logic Based Systems, Membership Functions, Simple and Parallel Big Bang-Big Crunch Optimization Approaches

1. INTRODUCTION

It is a well known fact that skilled and trained knowledge force is the most important requirement of present society. The nations with higher quality knowledge force will lead future world. The institutions of higher learning like universities or professional colleges play a very important role in producing this knowledge equipped man power. Increasing the number of these institutes will have a negative impact on the development if these miss the target of quality education and research. In order to improve the quality of education provided by these institutions it is imperative that the performance of these institutions to be assessed evaluated and improved continuously. Performance evaluation of universities and the institutes of higher learning is essential to identify, analyze and to remove the performance bottlenecks. Once the weak areas are identified and efforts can be focused for removing the weaknesses and academic excellence follows.

The institutes of higher learning are complex systems as large numbers of parameters directly or indirectly affect their academic performance. Further in addition to being large these systems are quite nonlinear as well. The classical techniques and the exact reasoning based approaches makes the design of performance evaluation system a very tough task. Such problems turn out to be NP Hard problems. Hence, one has to adopt approximate reasoning/ soft computing based approaches to design and implement such systems. Fuzzy logic based systems are one of the important class of knowledge based systems which simplifies the design of such type of complex systems. Zadeh gave the principles of fuzzy modeling [4]. Kumar, S. [5] introduced the principals and concepts involved in the design of fuzzy systems and explained

how such systems provides simple way to draw definite conclusions from vague, imprecise and incomplete information. Using fuzzy logic based approach, simplifies the design problem only to certain extent. Design of such system is a quite tedious and time consuming task as it needs large number of parameters to be considered which are difficult to measure. Further designing such system by eliciting knowledge from the domain experts is very difficult and compounds to system design progress. Discussions and interviews with the experts and design engineers are boring, cumbersome, time consuming and adds to the cost of the project. Thus designing of fuzzy system directly from the input and output data available is highly desirable. This paper presents an optimization method for identification of fuzzy logic based model from the numerical data available. Takagi, T et al [6] and Sugeno, M, et al,[7] in their papers gave the approach for building and tuning of fuzzy rules from the training data available. Wang and Mendel, in [8] and Mendel et al. [9] provided the rulebase generation and formulation of complete fuzzy system as two different problems. For evolving systems from available data many artificial neural networks based and fuzzy logic based approaches are available in literature. Neural network based approaches [10]-[13], GAs [14]-[21], ACO [22]-[28], BBO [29, 30] and PSO based approaches [31]-[33], for generation of rulebase and identification of fuzzy logic based system can be found.

Many researchers applied BB-BC and Firefly (FA) based optimization approaches for identification of fuzzy systems. Yang, X.S discussed the Firefly optimization algorithm for design of different systems.[34]-[36] Shakti Kumar et al in their paper [37] applied the FA optimization approach to two different system design problems and compared the performance with performance of other soft computing approaches.BB-BC optimization [2] have also been applied for fuzzy logic based model identification [38],[42]. Shakti Kumar et al. introduced multi-population BB-BC algorithm named parallel BB-BC Algorithm [3] and applied it to different design problems to compare its performance with other optimization algorithms [1],[3].

This paper presents a data driven approach to performance evaluation system design for the universities and institutes of higher learning using the optimization method which is based on behavior of fireflies called Firefly Optimization algorithm. FA is used to find out the optimal values of antecedent parameters and the consequents corresponding to each data set available and MSE is computed. With this approach such complex system can be designed with desired level of accuracy in a reasonable time. The results demonstrate the efficiency of FA designed system. The MSE of the system hence designed is compared with that of system designed using two other optimization approaches: simple and parallel BB-BC for the same set of data. Thus efficiency of all three approaches is compared to find out the most suitable approach.

This paper consists of 5 sections. Section 2 of this paper introduces fuzzy model identification for TSK Type-0 fuzzy system. In Section 3, a basic firefly algorithm and model identification based on this approach has been discussed. Section 4 discusses the proposed fuzzy model of institute rating system (IRS). Section 5 gives simulation results and compares the performances of three systems and finally next Section 6 concludes the paper.

2. FUZZY MODEL IDENTIFICATION FOR TSK TYPE-0 FUZZY SYSTEM

Fuzzy model identification is a process of designing the complete system from a given set of data. This fuzzy model identification process can be divided into three sub-processes namely Structure Specifications, Parameter Estimation and Model Validations [43]. Structure Specifications deals with input variable selections, partitioning of input spaces, membership function specifications and deciding the rule base of the system. In order to model a fuzzy model from a given training data set we proceed to formulate the problem as given below:

- Construct a fuzzy model with arbitrary selection of membership functions of given shapes for each input and output variable.
- Deciding the rule base for the fuzzy model.
- For entire training data set:
 - Evaluate output of the model for each training example.
 - Calculate error between the computed output and given output of the training example.
 - Compute mean square error (MSE) for the identified model.
- Minimize the objective/fitness function i.e. MSE using some efficient techniques.

Computing Output and MSE for Each Individual

For computing the MSE, both the actual output and the computed output of each individual is observed for all the training data points and error is calculated as per the following eq (1)

$$\text{Error} = O_A - O_C \tag{1}$$

Where O_C is

$$\text{Computed output } (O_C) = \frac{\sum_{k=1}^R w_k (R_k C)}{\sum_{k=1}^R w_k} \tag{2}$$

w_k is the firing strength of the k^{th} rule and $R_k C$ is the consequent of k^{th} rule.

And $O_A =$ Actual output as given in training data set

For all training data points MSE is computed. This gives the MSE of each individual, which in turn is used as the fitness function for rating the fuzzy model.

Minimize Objective Function (MSE)

$$\text{MSE} = \frac{1}{N} \sum_{k=1}^N [O_A - O_C]^2 \tag{3}$$

Subject to the Constraint That

- $R_k C \in \{\text{universe of discourse of output variable}\};$ (4)
- $X_{n \min} < E_{n1} < E_{n2} < \dots < E_{nm_n} < X_{n \max}$ (5)

Where O_A is the actual output, O_C is the computed output, N is number of data points taken for model validation and $R_k C$ represent consequent of k^{th} rule.

Thus this problem of fuzzy model identification from the given data is formulated as search and minimization problem. The optimization algorithm used must simultaneously adjust membership function parameters and consequents in

such a way so as to minimize the objective function i.e. MSE. In this paper we have applied Firefly based optimization approach to develop a suitable fuzzy model from the available training data set. Values of all the parameters of input and output variables such as membership functions and their shapes, along with consequents for the each rule were identified for the designing of complete fuzzy logic based system.

3. METHODOLOGY FOR FUZZY MODEL IDENTIFICATION FOR TSK TYPE-0 FUZZY SYSTEMS THROUGH FA BASED APPROACH

1.1 Firefly Algorithm (FA)

The Firefly Algorithm [37] is a nature-inspired, optimization meta-heuristic algorithm which is based on the social (flashing) behavior of fireflies. The primary purpose for a firefly's flash is to act as a signal system to attract other fireflies. Some of the flashing characteristics of fireflies can be idealized so as to develop firefly-inspired algorithms. For simplicity, the flashing characteristics of fireflies are idealized in following three rules [34]-[36]:

- All fireflies are unisex, so that one firefly is attracted to other fireflies regardless of their sex.
- Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly.
- The brightness of a firefly is affected or determined by the landscape of the objective function to be optimized.
- In the firefly algorithm, there are two important issues: the variation of light intensity and formulation of the attractiveness. For simplicity, it is assumed that the attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function.
- **Attractiveness:** The form of attractiveness function of a firefly is the following monotonically decreasing function [34]:

$$\beta(r) = \beta_0 e^{-\gamma r^m} \quad (m \geq 1) \quad (6)$$

Where r is the distance between any two fireflies, β_0 is the attractiveness at $r = 0$ and γ is a fixed light absorption coefficient.

- **Distance:** The distance between any two fireflies i and j at X_i and X_j , respectively, is the Cartesian distance as follows [34],[35]:

$$r_{ij} = \|X_i - X_j\| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \quad (7)$$

Where $x_{i,k}$ is the k^{th} component of the spatial coordinate X_i of i^{th} firefly and d is the number of dimensions.

- **Movement:** The movement of a firefly i is attracted to another more attractive (brighter) firefly j is determined by following equation:

$$X_i = X_i + \beta_0 e^{-\gamma_{ij}^2} (X_j - X_i) + \alpha (rand - 0.5) \tag{8}$$

Where the second term is due to the attraction while the third term is randomization with α being the randomization parameter. Rand is a random number generator uniformly distributed in [0, 1].

For most cases in the implementation, $\beta_0 = 1$ and $\alpha \in [0,1]$.

Parameters of FA

Light absorption coefficient (γ)	Number of fireflies (nf)
Randomization parameter (α)	Maximum iterations
Attractiveness factor (β)	

1.2 Methodology for Fuzzy Model Identification using FA

The steps to implement FA approach to fuzzy model identification are as given below:

Step 1 Initial Parameters Setting of Membership functions

Initially set the parameters of membership functions randomly.

Step 2 Initialize the FA parameters (as given in Section 3.1). Generate a random set of fireflies (initial population). Each firefly for this case is a system represented as shown in Figure 1.

Step 3 Apply a set of constraints which must be followed by set of fireflies.

Step 4 Build fuzzy model corresponding to each firefly.

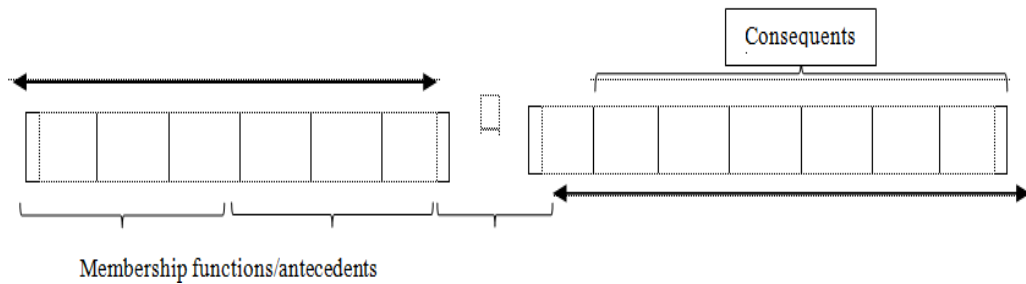


Figure 1: Representation of a Sugeno Fuzzy Model by One Firefly

Step 5 Evaluate each fuzzy model and calculate MSE (fitness function) for each model over entire set of training examples using Equation (3).

Step 6 Determine new positions of fireflies using Equation(s) (6), (7) and (8).

Step 7 If acceptable solution found then go to Step 9.

Step 8 If number of iterations NOT over then go to Step 3 for the next iteration.

Step 9 Stop

4. PROPOSED FUZZY LOGIC BASED MODEL FOR INSTITUTE RATING SYSTEM (IRS) [1]

In this section a fuzzy based system for the evaluation of institutions of higher learning is designed using Fire Fly Optimization approach. For designing such system all input and output variable parameters such as membership functions and their shapes, along with consequent for the each rule are identified. In doing so first inputs were decided then the shapes of Membership functions of inputs were fixed to be either triangular, trapezoidal or variation of these leading to sigmoidal or Z-type membership functions.

Shown below in figure 2, the block diagram of the desired multi input single output fuzzy system with $n= x_1, x_2, \dots, x_n$ number of input variables and $m= m_1, m_2, \dots, m_n$ number of membership functions for each input variable respectively. The system under design (IRS) [1] is having 14 input variables and one output variable named ‘‘overall rating’’ of the institution.

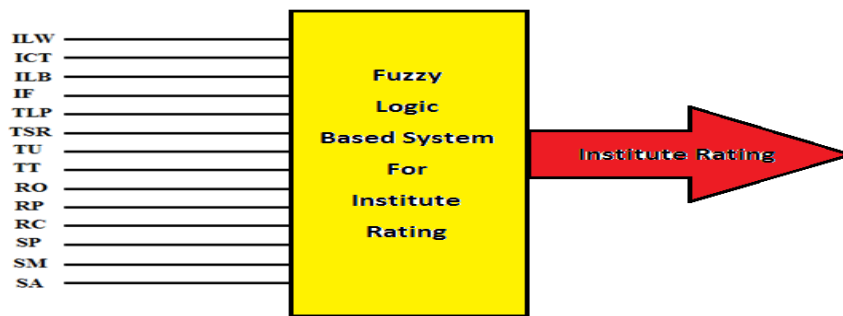


Figure 2: Block Diagram of the Proposed for Institute Rating Fuzzy System (IRS)

In this problem for each of these inputs number of membership functions are taken as 4($m=4$) and the consequents are selected from a range of 0.1 - 10. The shapes of membership functions are fixed as triangular membership functions and z-type membership functions for both input and output variables and placed symmetrically over the universes of discourse. First and last membership functions of each input and output variable are represented with z-type and sigmoidal membership functions respectively. All other membership functions are taken as triangular shaped. Shape of the four membership functions of the first input variable named Labs and Workshops (ILW) is shown in figure (3). The vertices of these fuzzy membership functions of the inputs are denoted as $E_{1,1} E_{1,2} \dots E_{1,4}, \dots E_{14,4}$. e.g $E_{1,1}$ means First input, first membership function and $E_{n,m}$ means n^{th} input, m^{th} membership function. Input parameters for the first variable $n=1$ are fixed such that: $X_{1,min} < E_{1,1} < E_{1,2} < E_{1,3} < E_{1,4} < X_{1,max}$ and overlapping of the adjacent membership functions is ensured.

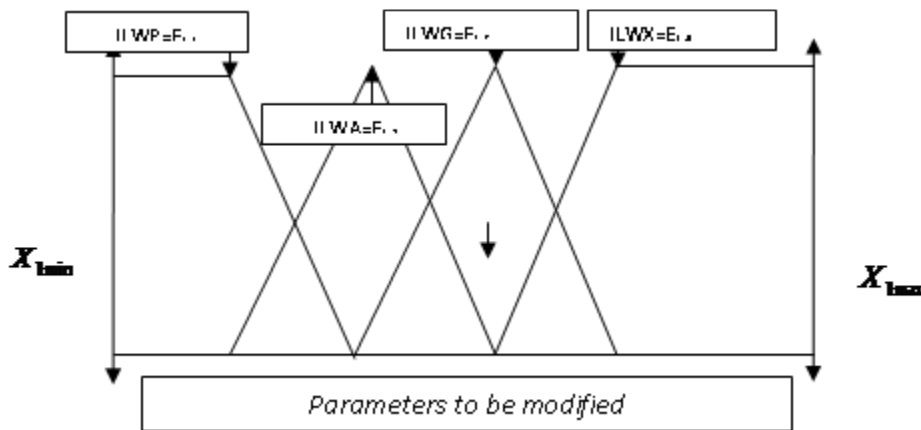


Figure 3: Membership Functions of one Input Variable [1]

Similarly all the 14 inputs are having 4 membership functions with similar shapes and values.

The total number of input parameters (vertices) and the total number of rules are given by the Equation (9) and equation (10) respectively.

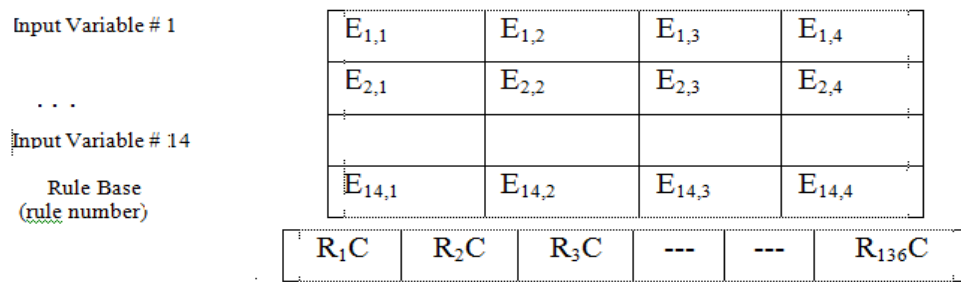
$$\text{Total number of membership functions} = \sum_{j=1}^n m_j \tag{9}$$

$$\text{Total number of rules (second constituent part of individual)} = \prod_{l=1}^n m_l \tag{10}$$

$$\text{Thus Size of one individual (Sugeno model)} = \sum_{j=1}^n m_j + \prod_{l=1}^n m_l \tag{11}$$

In this model we have considered 14 inputs each with 4 membership functions and a set of 136 rules only. Thus the total size of the individual is calculated as per the Eq (11)

$$\text{Individual size} = \text{input parameters} + \text{consequents} = 56 + 136 = 192$$



$R_1 C$: Consequent of Rule1; $R_2 C$: Consequent of Rule2; and so on

Figure 4: Representation of a Sugeno Fuzzy Model by One Individual

The individual shown above in the figure (4) is a complete fuzzy system whose different parameter values are modified randomly to find out the best suited system with desired results. With this methodology system can be designed for any number of inputs with any number of membership functions. Movement of the membership functions is given as per the following

For ensuring a movement of membership functions to right, we use the following equations:

$$E_{ni} = E_{ni} + (E_{n(i+1)} - E_{ni}) * P_k \tag{12}$$

Where $i=1, 2, \dots, mn, k=1, 2, \dots, etc.$

$$\text{If } (i = m_n), \text{ then } E_{ni} = E_{ni} + (X_{n \max} - E_{ni}) * P_k \tag{13}$$

Here P_k decides the percentage of movement.

For the movement of membership functions to left, we use the following equations:

$$E_{ni} = E_{ni} - (E_{ni} - E_{n(i-1)}) * P_k \tag{14}$$

$$\text{If } (i = 1), \text{ then } E_{ni} = E_{ni} - (E_{ni} - X_{n \min}) * P_k \tag{15}$$

We identified this fuzzy model using firefly optimization based approach. The algorithm is implemented in MATLAB. In this case a population of N individuals is first randomly generated. Each individual in the population represents a complete fuzzy system which consists of two parts: first part represents membership functions of antecedents and the second part represents rule-base. To obtain the optimal solution the membership functions and rule base are modified simultaneously.

5. SIMULATION RESULTS

In order to validate our approach of system identification we conducted 10 sets each consisting of 10 trials and recorded the MSE for each of the evolved system. The iterations versus MSE graph for one of the trials with firefly algorithm based optimization approaches given in figure (5). We ran the program for 140 iterations. This trial run produced a minimum MSE= 0(zero) in 284.333768 seconds.

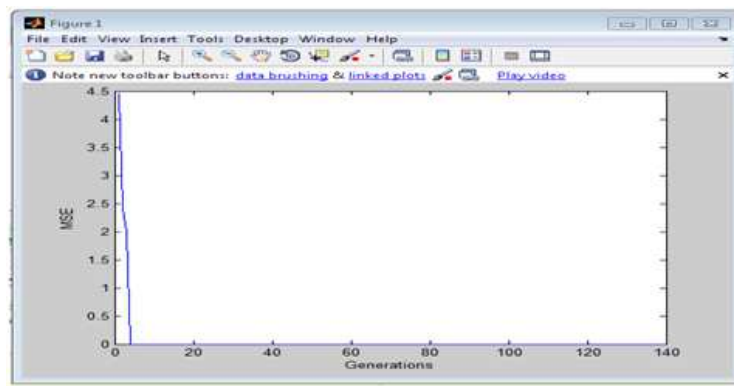


Figure 5: Iterations Vs.MSE for Firefly Algorithm Based Approach

Figure (6) and (7) give the Iterations Vs.MSE for simple and parallel BB-BC based approaches

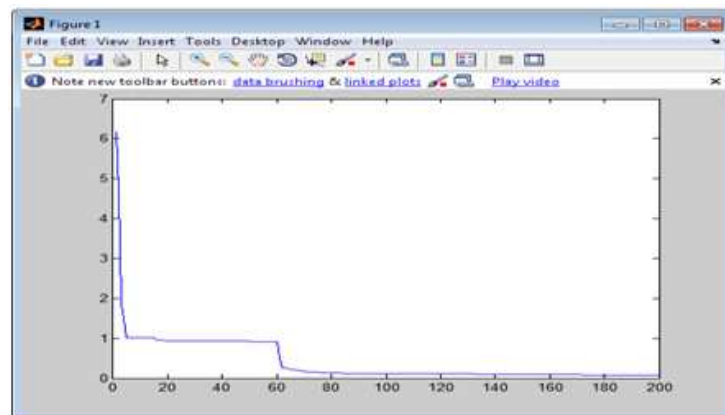


Figure 6: Iterations Vs.MSE for simple BB-BC based approach [1]

As shown in the figure (6) system identification with simple BB-BC approach produced MSE of 0.0761937 in 230.391967 seconds. In these trials the program was executed for 200 iterations. Figure (7) presents the results for one of the trials for parallel BB-BC based approach. Parallel BB-BC based approach gave the minimum MSE =0(zero) only in 64.678327 seconds. This MSE is

Achieved within 30 iterations in this trial

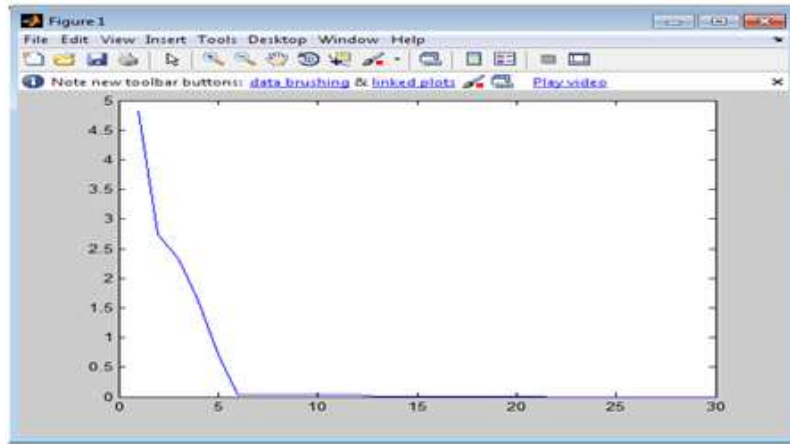


Figure 7: Iterations Vs.MSE for Parallel BB-BC Based Approach [1]

Comparison of the Performance of Simple BB-BC, Parallel BB-BC and Firefly Based Algorithms

Table 1 given below presents the performance comparison of three approaches. It is evident from the table that whereas minimum, average and maximum MSE with simple BB-BC was observed to be 00.00, 0.0505 and 0.1861, the minimum, average and maximum MSE for parallel BB-BC approach and Firefly optimization approach was observed to be 00.00 (zero). Thus both the approaches produced better results than simple BB-BC approach. But when we compared the execution time parallel BB-BC approach produced zero MSE only in average time of 64.68 seconds where as firefly approach took average time of 286.11 seconds for producing same result.

Table 1: Simple BB-BC, Parallel BB-BC and Firefly Performance Comparison

Optimization Approach (nind/Iterations)	MSE			Execution Time (sec) for min MSE		
	Minimum	Average	Maximum	Worst	Average	Best
Simple BB-BC(40/200)	00	0.0505	0.1861	328	236.18	124.98
Parallel BB-BC(15/28)	00	00	00	63.9	64.68	66.1
Firefly App(80/140)	00	00	00	289.7	286.11	281.8

6. CONCLUSIONS

This paper presented Firefly Optimization Approach for the identification of the system and compared the results with the multi-population parallel BB-BC and simple BB-BC approaches. We applied this approach to identify a 14 input- single output fuzzy logic based system for evaluating the over-all rating of universities and institutes of higher learning. Each input variable consists of 4 membership functions. A canonical system could have consisted of $4^{14} = 268435456$ rules. This rule explosion makes it difficult to identify a complete rating assignment system using knowledge driven approach. We used the available 136 point training data to identify a fuzzy logic based system with 136 rules. We evolved the models using Firefly optimization approach, simple BB-BC as well as parallel BB-BC approaches. We conducted model identification experiment for 10 sets each of 10 trials. Whereas for training data, Firefly optimization approach identified the system with MSE zero in time 286.11 seconds, simple BB-BC identified best model with average MSE of 0.0505 in average time of 236.18seconds and parallel BB-BC identified the model with zero MSE in average time of 64.68 seconds.

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